

Memories of Math: Visitors' Experiences in an Exhibition about Calculus



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ABSTRACT

Handling Calculus is a set of interactive exhibits about mathematics developed by the Science Museum of Minnesota and TERC. The exhibits are designed to engage visitors with kinesthetic, application-oriented, and concept-focused approaches to calculus and pre-calculus concepts. As we examined visitor interviews collected during the evaluation of *Handling Calculus*, it was striking how often respondents' experiences with the exhibits stimulated memories they associated with school math. Respondents recognized math terms and symbols, reconstructed graphing skills, and recalled fragmentary concepts they had learned in school. In addition, they recounted stories about their former math teachers and the ways in which they had been taught mathematics, as well as a range of both positive and negative emotions they associated with school math. Respondents recognized and valued the hands-on and whole-body approaches used in *Handling Calculus*, contrasting the exhibits with their paper-and-pencil school experiences. Some respondents wondered if they would have done better at school calculus if their teachers had used the approaches taken by *Handling Calculus*.

INTRODUCTION

If you don't like math, you may be thinking, "Why would any visitor, other than an engineer or scientist, willingly enter an exhibition about calculus?" Here is one answer: Learning calculus does not have to begin with thick textbooks, droning teaching assistants, and red-ink-stained homework. Instead, calculus can begin with hands-on and full-body experiences like those that visitors have long enjoyed in science centers around the world.

The Science Museum of Minnesota and TERC¹ created a series of interactive exhibits about calculus and pre-calculus concepts, most of which involved graphing visitors' actions in real time on computer monitors and analyzing the resulting graphs. For instance, when visitors moved their bodies back and forth in front of a motion sensor, a computer continuously observed their location and plotted their position in time. When visitors slid cutout characters to tell a story, manipulated a graph to plan trips, or poured running

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water into a row of plastic boxes, computers graphed those actions, as well. Once visitors made graphs, those graphs could be used to demonstrate key concepts of calculus.

Evaluation played a major role in the *Handling Calculus* project. Our overall approach to these studies included detailed observations of visitors as they used the exhibits and loosely structured, in-depth interviews with visitors as they prepared to leave the exhibition (an approach we call naturalistic inquiry). Evaluation took place in three phases. First, individual exhibit units were tested with visitors and then revised until they more effectively accomplished their goals (formative phase). Next, we tested the effectiveness of a trial version of the whole exhibition with visitors on the museum floor, and again the exhibition was revised based on the results (remedial phase). Finally, the overall effectiveness of the completed exhibition was assessed (summative phase). This paper is based on a re-examination of data collected during the last two phases. During this re-analysis, we focused on understanding visitors, rather than judging the effectiveness of the exhibitions. (See Appendix A for a more complete description of the evaluations.)

So, although this paper is based on findings from the *Handling Calculus* evaluations, it is not a report about the evaluations, and it is not primarily about the exhibition. Instead, this article is about visitors. It shares some findings about visitors and what happened to them as they engaged with the exhibits. Speaking metaphorically, we pulled threads that ran through the remedial and summative evaluations, wove them into a thematic tapestry, then cut and sewed that tapestry into the linear quilt that is this paper. With a different selection of threads, a different weaver, or another seamstress, the quilt—and paper—might look very different, emphasizing different points and drawing different conclusions. This paper presents some of the themes that the exhibit team and author found most interesting about our results: how Science Museum visitors' memories and feelings about math shaped their experiences in *Handling Calculus*, and what that might mean for future exhibitions about math. We hope these findings will be useful for developers of future exhibitions about mathematics, especially those about higher math.

ABOUT HANDLING CALCULUS

In their National Science Foundation (NSF) grant proposal, the project team postulated that: “The kinesthetic experience of physical actions such as motion, shape, and flow can make the ideas of calculus accessible and illuminating to a general audience, including children in middle school and younger.” They set out to realize this statement by developing a series of exhibits focused on key concepts of calculus, like graphs of functions, slope, derivatives, integrals, and limits. Most exhibits included both a computerized interactive experience and more detailed explanations of the calculus in associated label panels. The interactive experiences began with visitors doing something related to graphs; calculus and pre-calculus concepts came into play as visitors began to manipulate these graphs. The open-ended nature of the interactives allowed for multiple outcomes at each unit, depending on visitors' knowledge and experience and the ways in which they engaged with specific units.

Three exhibits explored the making and interpretation of motion graphs and the

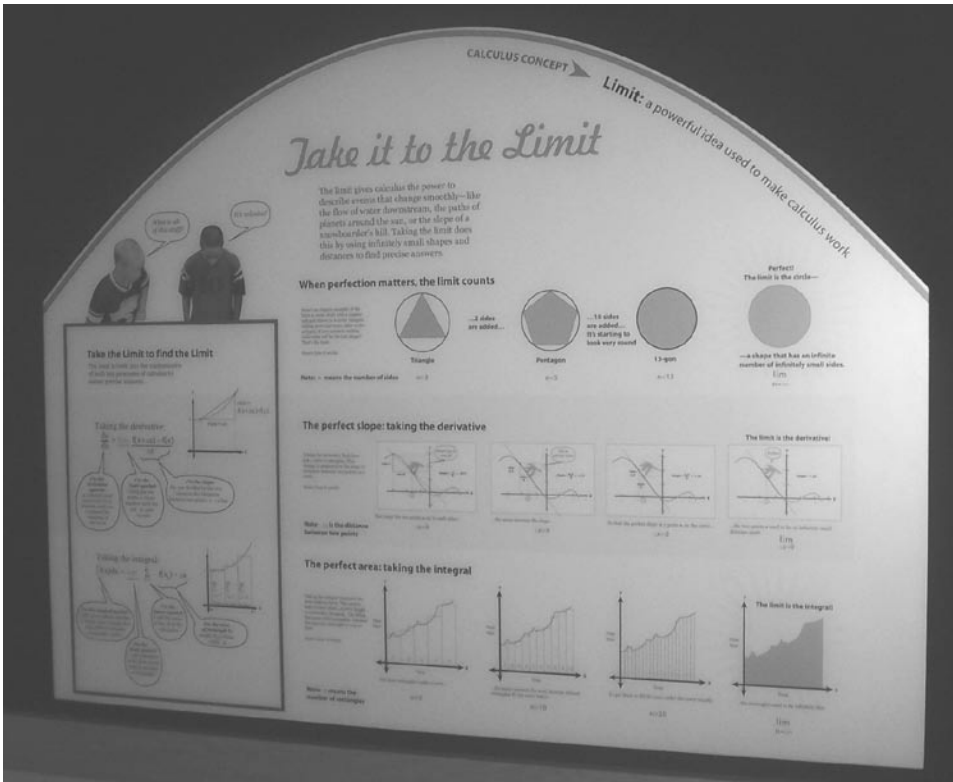


The Archimedes Limit interactive. Exhibits on the concept of limits, in a large gazebo containing a laser and circular mirror, which created a sequence of shapes with increasing numbers of sides, eventually making a circle. *Photo by Eric D. Gyllenhaal.*

calculus concept of differentiation. At Motion Tracker, Math Tracks, and Road Trip, visitors moved things—their bodies, sliders, or a simulated car—and watched the results graphed on a computer monitor as changes in position over time. Visitors could choose to graph the derivative of the position graph, which showed changes in velocity over time.

Slope Rider allowed visitors to explore the concept of slope, calculated as “rise over run,” a key to understanding differentiation. At the Flow Integrator, visitors controlled the position of a stream of water over a series of vertical bins, then pushed a button to simulate the integration of the volume under a curve, as explained and illustrated in an adjacent label panel.

The only major exhibit not based on visitors’ actions was Archimedes’ Limit, in the center of a gazebo. Here visitors watched as a mechanized laser used fog and mirrors to create a sequence of shapes with increasing numbers of sides—first a triangle, then a square, pentagon, hexagon, and so on—until the “infinite” number of sides made the shape resemble a circle. The Take It to the Limits label panel, on the back wall of the gazebo, illustrated the relationship between the laser-light shapes, limits, and the calculus procedures of differentiation and integration. (All of these exhibits are discussed in more detail in Appendix B.)



Take It to the Limit label from the Archimedes Limit gazebo. With text and graphics, this panel related the multi-sided laser-light shape to basic calculus concepts, derivatives and integration. Photo by Eric D. Gyllenhaal.

VISITORS' FEELINGS ABOUT MATH AND CALCULUS

As we talked with adult visitors, we were initially surprised at how many had taken calculus in either high school or college. However, the literature indicates that about 40 percent of American undergraduates take at least one calculus course (Wilson 1997). Given the preponderance of college graduates in science center audiences, we should not be surprised that so many adult visitors had studied calculus at some point in their lives.

Visitors' feelings about school math—Some have portrayed mathematics as a cold, analytical, and even emotionless subject, but that view did not fit with our respondents' experiences with school math. When visitors entered *Handling Calculus*, their latent feelings about mathematics—especially school math—usually emerged. Some visitors used vivid terms like “hate” to describe their difficult relationships with all school math, while others expressed more focused negative feelings, such as the respondent who informed us, “I liked algebra, and I hated geometry.” At the other extreme, we talked with visitors who confessed their love for school math and wished they could use math more in their post-

school lives. More typically, *Handling Calculus* respondents expressed ambivalent feelings about math, describing both their successes and difficulties at school.

While there is quite a bit of math education literature about students' math anxiety, math avoidance, and fear of math (see Tobias 1993; Zaslavsky 1994), the few studies we found characterizing post-college adults described a much broader range of feelings about math (Karsenty and Vinner 2000; McSheffrey 1992), much like what we encountered talking with visitors in *Handling Calculus*. In a study of adult Israelis with successful, mostly non-mathematical careers, Karsenty and Vinner (2000) classified their respondents into four mathematical self-schema: 1) The "positive schema" included those who associated feelings of pleasure, challenge and self-confidence with school math. 2) The "semi-positive schema" included those who respected mathematics and felt it was important, but who remembered working hard at school math while gaining little pleasure from their efforts. 3) The "indifferent schema" included those who said they were not enthusiastic about school math, but also were not frustrated by it—they simply went through the motions as required of them. 4) The "negative schema" included those who expressed their fear, resentment, stress, and frustration with school math. As adults, most negative-schema subjects "avoid any unnecessary contact with mathematics" (Karsenty and Vinner 2000, 3–123).

Reviewing our interviews with adult respondents in *Handling Calculus*, we found the Karsenty and Vinner classification a useful and fairly complete description of our adult respondents. In describing visitors for this paper, we use the term "math-averse visitors" for visitors who fit the negative schema, refer to positive-schema visitors as "math lovers," and use the terms "semi-positive visitors" and "indifferent visitors" for those who seem best classified between the two extremes. We found indications that the middle ground between the positive and negative poles might be more complex than implied in the Karsenty and Vinner classification. It was particularly interesting that, when confronted with *Handling Calculus*, some semi-positive and indifferent visitors seemed to be reconsidering their feelings about higher math.

Visitors' feelings about calculus—Some respondents spoke at length about their experiences with, and feelings about, calculus and pre-calculus courses they took in high school and college. Wilson (1997) found that 40 percent of current undergraduate calculus students were failing their courses, so we weren't surprised that many visitors to *Handling Calculus* described their difficulties and frustrations in calculus classrooms.

We noticed two trends in our data. First, although some visitors told us that they hated math or geometry, none of our respondents focused their math hatred specifically on calculus. Instead, when respondents discussed their negative feelings about calculus, they usually focused on their courses and teachers, especially those they had in college. Respondents expressed more positive feelings about their high-school calculus and pre-calculus teachers; some said they consulted high-school notes and textbooks to survive college calculus. Second, to an even greater degree than the rest of math, calculus seemed to our respondents to be about school, not about their everyday lives. Calculus was something that many respondents had once struggled with and learned to respect, but it wasn't something most adult respondents used in their jobs and lives today.

Who is the exhibition for?—Given that respondents didn't use calculus once they left school, we wondered if they considered *Handling Calculus* to be meant for other people, perhaps those who still used calculus. Indeed, some respondents indicated that the exhibits were aimed at the more mathematically talented members of their groups. For instance, one parent stood back and watched her middle-school-aged son, whom she called "the mathematician," working at the exhibits. We also watched an older couple return to the exhibition after a 20-minute absence; they had gone to find their college-age son and his girlfriend who were taking calculus, so they could see the exhibits. When we asked another visitor if he wanted to choose the "Do the Calculus" option at an exhibit, he quipped, "Where's Katie when we need her?" It turned out that Katie was a mathematically talented cousin. Although these visitors said they appreciated the exhibits, it was clear they felt that *Handling Calculus* was most appropriate for other, often younger members of their groups.

In contrast, we talked with many respondents who took ownership of both the exhibits and at least some of the concepts portrayed in them. Even children and adults who had never taken calculus felt some exhibits were for them when they recognized familiar features like graphs and multi-sided shapes or dimly remembered concepts like summing up, rise over run, and infinity. Because they remembered these ideas or still used them, these respondents found starting points for their use of the exhibits.

Respondents who were currently calculus or pre-calculus students often expressed their excitement about *Handling Calculus* because the exhibits approached concepts they were studying in school in ways that seemed new and fun to them. Although a few current students were clearly not interested in engaging with the exhibits, most took the opportunity to explore differentiation and integration in a hands-on or whole-body way. Among adults who had taken calculus years or even decades before, math lovers who had been successful in college calculus often expressed their appreciation for the exhibits. *Handling Calculus* allowed them to relive and share with their companions a subject they once had enjoyed but almost never thought about today. Most interestingly, some semi-positive adults who had suffered through college calculus also appreciated revisiting the subject. They sometimes took *Handling Calculus* as an opportunity to reflect on what went wrong with their earlier calculus educations. A few semi-positive and indifferent respondents said things like, "If I had the time and money, I would like to go back and try calculus again." We return to this theme later in this paper.

MATH-AVERSE VISITORS IN *HANDLING CALCULUS*

We spoke with quite a few math-averse visitors, including self-described "math haters" who, one way or another, found themselves in *Handling Calculus*. A few math-averse visitors told us their companions had dragged them into the exhibits under protest. Others said they saw people doing things that looked interesting from a distance, only to find themselves surrounded by math exhibits once they crossed into *Handling Calculus*. A few visitors told us that they had wandered in more or less by chance, not paying much attention to what they were doing, or just following their young children.

Once they realized the exhibits were about math, math-averse visitors used a variety of strategies to cope and perhaps find some enjoyment—or to minimize their pain. We identified five common strategies used by math-averse respondents:

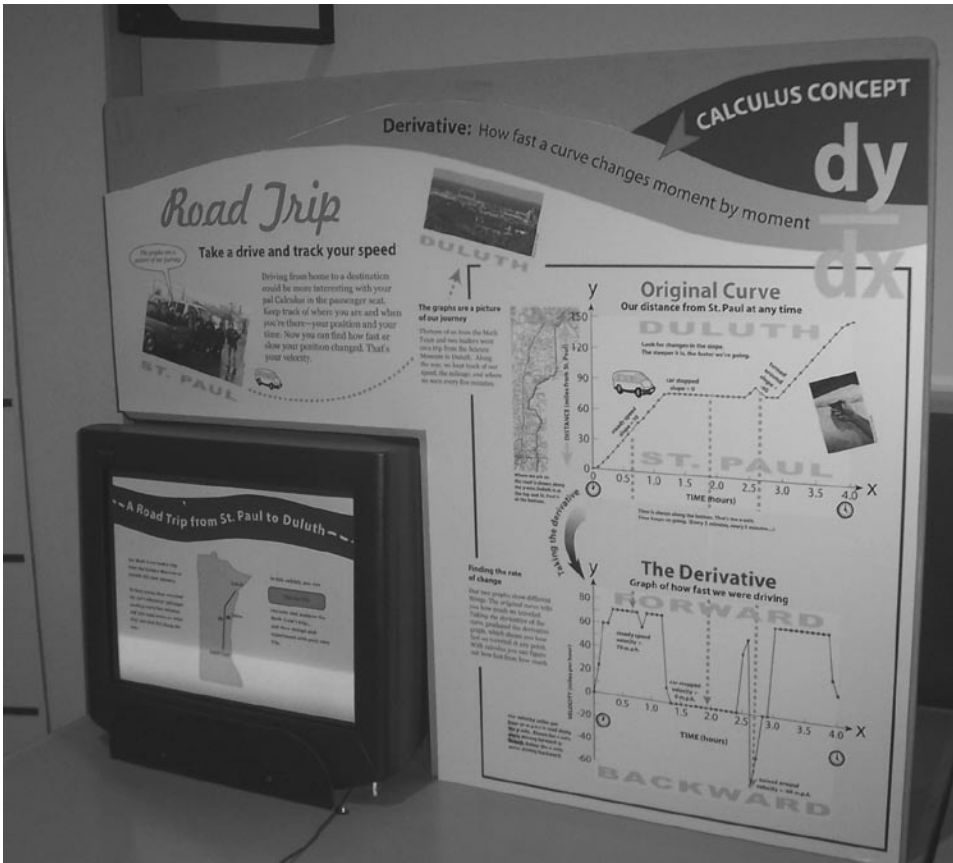
Stand and wait—When math-averse visitors were dragged into *Handling Calculus* by others in their group, they sometimes refused to engage with the exhibits. Instead, they stood to the side and waited for their companions to finish—sometimes patiently, sometimes sulking and complaining.

Look and leave—Other math-averse visitors looked around *Handling Calculus* for a bit and then left without engaging with the exhibits to any great extent. For instance, once one respondent figured out that Road Trip did not give her any interesting details about the journey from St. Paul to Duluth, she exclaimed, “It’s just a graph!” and left the area.

Avoid and engage—Once math-averse visitors entered *Handling Calculus*, it was not unusual for them to engage just with the aspects of the exhibits that they felt were the most fun, or least challenging. These visitors often said they ignored adjacent labels with calculus words, graphs, and equations, especially when the exhibits presented relatively simple challenges, like using a laser to hit a target at Slope Rider or filling containers with just the right amount of floodwater at Flow Integrator. These visitors sometimes disengaged completely from an exhibit once they reached a step that seemed difficult to complete, especially if it included a mathematical procedure or concept. For instance, when we asked a current calculus student why she chose not to press the Road Trip computer’s button labeled “Take the Derivative,” she said, “I got a C- in calculus, so when I saw derivatives, I tried to avoid that part.”

Social influences—Other visitors who disliked math were drawn into deeper levels of engagement through interactions with their companions. Some adult visitors let a more mathematically-accomplished member of their group take them through the exhibits, showing them what to do and trying to explain the more difficult mathematical concepts. The most successful exhibits for these teaching-learning dyads included labels with clear, relatively simple language and graphics that facilitated the teacher’s efforts. Some parents who disliked math nonetheless took on a teacher role for their elementary-aged children. These parents often said their children were more talented at math than they themselves were, and they wanted to help their children enjoy and learn from the exhibits. Exhibits facilitated these interactions when parents could quickly find and read simple explanatory text and graphics. For instance, at Motion Tracker, we watched a mother give five- and six-year-old children detailed instructions so that they were able to do a pretty good job matching the graphs. When we interviewed her, she said, “I have no idea what I was doing; I just read the directions.”

Using humor—A few math-averse visitors coped by making fun of exhibits. At Road Trip, one visitor enjoyed making fun of a label that read, “Driving from home to a destination can be more interesting with your pal Calculus in the passenger seat.” This defensive use



In the Road Trip interactive, visitors make trips to other cities. Graphs on the monitor show a distance vs. time plot of the trip. The derivative of a graph shows how fast their car was traveling at any time. *Photo by Eric D. Gyllenhaal.*

of humor seemed to help him get past his negative feelings and sit down at the computer. His group proceeded to make fun of many other aspects of the program, including getting stopped for speeding in Montana at only 70 miles an hour.

While some math-averse visitors did not enjoy or take full advantage of *Handling Calculus*, others found themselves engaged with the exhibits despite their dislike of math. When considering why that happened, three things stand out. First, many visitors responded to the simpler challenges presented by the exhibits, and they tended to stay engaged when they experienced success. By helping visitors find appropriate challenges with accompanying success, certain exhibits were able to hook more visitors (Perry 1989). Second, many visitors responded well to teaching-learning interactions within their social groups. We found that labels that modeled and facilitated the teachers' roles improved the effectiveness of the exhibits. Finally, some visitors were clearly hooked by humor. Labels might be more effective if they help visitors laugh about their math experiences.

When determining the appropriate level of challenge for math exhibits, it is important to balance challenge with success. During the earlier stages of the evaluation, we often encountered visitors who gave up in frustration because they did not understand what to do, especially when they had to follow a complex series of steps. Math-averse visitors in particular seemed to give up quickly, after following just a few steps in the procedure. On the other hand, visitors who liked math sometimes spent several minutes trying to figure out confusing exhibits, often without success; in fact, during early stages of prototype testing, it seemed that math lovers suffered more for their deeper interest. As prototyping proceeded and developers shortened the sequences of steps and clarified the instructions, both math lovers and the math averse were more successful and less frustrated. It was clear that iteratively testing and revising exhibits can make them more effective for all visitors, but prototype testing is vital to effectively serve the math averse.

As developers consider how to facilitate teaching-learning interactions within groups, they must remember that parents often function as teachers even when they know little about the topic of the exhibit (Perry 1989). In fact, this was a major problem in *Handling Calculus*. As children punched the touch screen at Slope Rider, moved the sliders at Math Tracks, or poured water at Flow Integrator, their adults often read enough to realize the exhibits were about calculus. Taken aback, they often could not find words to explain the exhibits to their children. One visitor read a label, told his seven-year-old nephew, “This is calculus, James,” and walked away. However, parents and adult caregivers can function as effective teachers when they find appropriate labels. Perhaps the best example in *Handling Calculus* was Archimedes Limit. When parents found the graphic that showed a sequence of silhouetted shapes, they were able to help even six- and seven-year-old children understand how a triangle can evolve into a circle. This led to prolonged engagements for whole families at this exhibit.

“NESTED” CONCEPTS IN *HANDLING CALCULUS*

During previous evaluations of other exhibits in the Science Museum’s Experiment Gallery, we found that exhibits about sophisticated concepts were particularly effective for educationally diverse audiences when they provided rich experiences that were interpreted and understood on a range of levels (Garibay and Gyllenhaal 2003a). When the ideas learned by younger and less sophisticated visitors were prerequisites to the concepts that more sophisticated visitors took away from the exhibits, we referred to these exhibits as embodying “nested concepts” (Gyllenhaal, Cheng, and Carroll 2004).

There were indications that several exhibits in *Handling Calculus* helped visitors find and work with a range of nested concepts. For instance, at Flow Integrator, younger children explored the properties of running water as they mastered the pouring mechanism; older children matched their pouring efforts to the graphs and then summed up the result; and visitors who had taken calculus sometimes recognized the process as an application of integral calculus to a real-world problem. In the Archimedes Limit gazebo, younger children counted the sides as the laser figure evolved toward a circle with infinite sides; older children and adults contemplated their encounter with the infinite; and

visitors who had taken calculus sometimes revisited the key concept of limits using the adjacent label panels. The simpler ideas that children and less sophisticated adults took from the exhibits were “nested” because they were prerequisites for developing a gut-level understanding of what calculus is and does.

HOW VISITORS’ MEMORIES OF SCHOOL MATH SHAPED THEIR EXPERIENCES

In part because of this nesting of concepts, most children and adult respondents related their *Handling Calculus* experiences to the math they were doing, or had done, in school—and sometimes took their understanding a step or two further. This process of connecting to school math seemed an important step for making sense of the exhibits, at least for older children and adults. In the museum field, we often talk about the importance of establishing personal connections between visitors and museum exhibits. In this case, school math was that personal connection for many visitors. Perhaps that was because so much of people’s experience with math beyond arithmetic takes place in school settings.

Recognizing calculus—One striking finding in the *Handling Calculus* evaluations was that visitors who lacked calculus experience rarely identified the exhibits as being about calculus; instead, they described them as being about math in general, or about topics specific to individual exhibits, like graphing, rise over run, or infinity. Respondents who described the exhibits as being about calculus almost invariably had some prior knowledge of the subject. These included adults who had studied calculus in high school or college, current students in calculus or pre-calculus, and younger students who had heard about calculus from their parents or older siblings.

Use calculus, or lose it—Most respondents who had taken calculus years ago claimed they had forgotten everything they learned. Respondents often said they had never used calculus after they left college, and, as one respondent claimed, “If you don’t use it, you lose it.” However, as they used the exhibits, these same visitors often recognized some calculus words, symbols, or ideas, and could distinguish them from topics they did not remember studying. Respondents described how they vaguely remembered some concepts (like calculus being about change) and phrases (like “taking the area under the curve”), but did not remember how to apply them within the exhibit. As one such visitor said, “If you haven’t used a skill, you do lose it. You may remember bits and pieces, but if you were to ask someone years after they’ve been in the class to do an integration, they would be going, ‘Hmm.’” Visitors’ “use it or lose it” perceptions of calculus resonated with a study by Garner and Garner (2001), who found that, less than a year after taking college calculus, most students forgot most of what they learned in the course—but not everything.

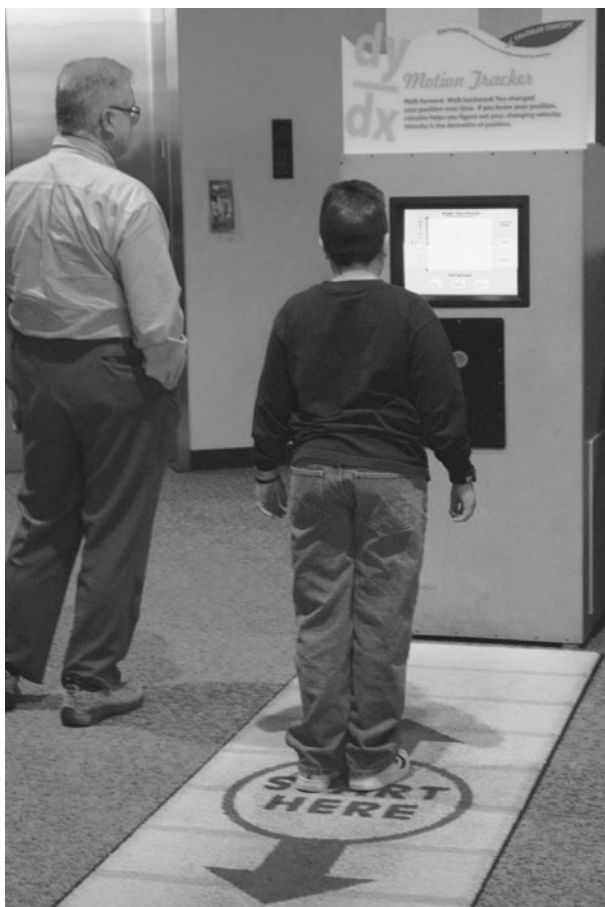
Applying math skills—Many *Handling Calculus* respondents were able to apply at least a few skills learned in school math, especially precursor concepts like graphing, slope, and aspects of algebra and geometry. This result resonates with a study by Bahrck and Hall (1991), who found that most adults remember at least some high school math concepts,



In the Math Tracks interactive, visitors create and interpret graphs of the motion of sliders on a track, re-enacting one of four scenarios based on stories, or creating freeform motion graphs in Free Exploration mode. *Photo by Craig Thiesen, courtesy of the Science Museum of Minnesota.*

and that those who took calculus in college remembered algebra and geometry much better than those who had not. Visitors who we classified as showing semi-positive, indifferent, and even negative schema were able to recognize and apply concepts precursor to calculus *when these ideas were expressed in an accessible way*. For instance, most adult respondents said they recognized the “rise-over-run” definition of slope and could apply it to the Slope Rider exhibit. Respondents said they remembered learning and using “rise-over-run” in a variety of math courses, including algebra, geometry, and statistics, although they almost never recalled using it in their post-school lives.

As visitors applied their fragmentary memories to the *Handling Calculus* exhibits, we were reminded of a qualitative study by Karsenty (2002). When Karsenty’s adult respondents were asked to sketch graphs of simple linear functions, their efforts suggested on-the-spot, sometimes idiosyncratic attempts to reconstruct what they had learned in school. Respondents used common sense to fill the gaps in their understanding, constructing a “new, but coherent, chain of ideas” (2002, 142) that linked their fragmentary memories. Because they took math long ago and were working without interpretive support, the resulting graphs were usually incorrect. However, science museum visitors—like those in



The Motion Tracker interactive. As visitors move on the rug (“start here”) a sensor determines their position continuously, and data are graphed on a computer monitor. *Photo by Craig Thiesen, courtesy of the Science Museum of Minnesota.*

Handling Calculus—recall and reconstruct mathematical ideas in a much more supportive setting. By helping visitors recall appropriate ideas and guiding them towards appropriate reconstructions of concepts they learned long ago, exhibit developers can maximize the value of visitors’ fragmentary memories of math.

WHAT VISITORS TOOK AWAY FROM THEIR EXPERIENCES

One area of interest for the *Handling Calculus* project was to identify what visitors took away from their experiences. The evaluations showed that learning outcomes varied greatly depending on visitors’ existing knowledge and feelings about math, the knowledge and backgrounds of their companions, and the exhibits with which they engaged.

It is not surprising that learning outcomes varied by age. Some visitors—especially elementary-aged children—learned new concepts about math, especially concepts that students encounter on the educational road to calculus, like interpreting motion graphs, measuring slope, and what happens as the number of sides of a polygon approaches infinity. A few deeply engaged middle-school children seemed to learn simple calculus con-

cepts as, for instance, they used graphs of the derivative to plan gasoline and meal stops and avoid speeding tickets. However, for many older children and adults, the *Handling Calculus* experience was more often about remembering and revisiting ideas they had encountered earlier in school math. These respondents described three different ways they valued their experiences in *Handling Calculus*.

A different way to experience math—The kinesthetic approach used by *Handling Calculus* was, to a large extent, the key to its success. Visitors had a more physical—and more fun—way to experience concepts and procedures that they had previously only read about and done on paper. Current calculus students realized they were experiencing a familiar subject in a new way, resulting in expressions like, “Oh, this is fun!” and “That’s so cool!” More thoughtful students explained that the hands-on approach complemented what they were learning in school. For instance, one college student said he liked these exhibits because he got to understand the concepts, whereas in class he had been so wrapped up in formulas and problem solving that he did not really understand the ideas behind them. He said he was reminded of things he had been exposed to before, but he was approaching them in a different way.

Sharing the experience—Those who knew more about math often shared their knowledge with others in their groups, usually to the benefit of both. In some cases, both teacher and student roles were filled by adults; in other cases parents worked with their own children. Sometimes the more knowledgeable visitors were parents who enjoyed helping their children succeed with exhibits about subjects they had enjoyed and succeeded with at school. Other times parents said they knew little about math, but when they found the right text and graphics they were able to guide their children nonetheless, staying a half step ahead and sometimes learning new ideas themselves. Such parents often saw their children as more talented at math than they were, and they appreciated the opportunity to help them—and sometimes to receive their children’s assistance.

A visit with an old friend—Although some visitors laughed at the idea of “your pal Calculus in the passenger seat,” others talked about *Handling Calculus* as if they were visiting an old, nearly abandoned friend. These respondents said they regretted never using calculus in their work, because calculus was a way of thinking and dealing with the world that they had enjoyed and profited from. They missed having it in their lives. Some respondents thanked the Science Museum for letting them spend time with calculus, saying things like, “For me it was a great refresher. I’m glad it’s in.” A few lamented that they would not have many more opportunities to use calculus in their lives.

In addition to valuing their *Handling Calculus* experience, some visitors shared with us unresolved conflicts about their relationships with college calculus still festering below the surface. These visitors exposed and began to resolve these conflicts in *Handling Calculus*, in at least three different ways.

Matching learning styles—The first type of resolution was based on insights about the kinesthetic approach to learning and how that related to visitors’ own learning styles. For in-

stance, after helping her children make and understand graphs at Math Tracks, one young mother exclaimed, “This is what I should have had when I took high school calculus!”

Applications before theory—A second but related type of resolution depended on the applications-first, theory-second approach taken by, for instance, the Flow Integrator. We talked with several visitors who said they found this approach more meaningful than the theory-first (or formula-first) calculus they had taken in college. In various ways, they told us that seeing how calculus was used helped it make sense to them: “I like seeing what it does.”

Clear explanations—The third type of resolution was facilitated by large graphic panels adjacent to most interactives. Although these labels were very complex by museum standards, by school-math standards they clearly summarized key concepts using effective combinations of text, symbols, and graphs. These panels were avoided or ignored by many visitors, and they tended to mystify visitors who merely glanced at them. However, we also watched adults study these panels carefully for up to 10 minutes, sometimes returning to them later to discuss them with their companions. It turned out that these visitors had taken calculus long ago, and when we asked them why they had spent so much time with one label, they said things like, “I wish someone had explained this to me when I was in college. Now it makes sense to me,” and “This makes total sense to me. I never got it before.” When we asked one respondent what she had learned reading the “Take it to the Limit” panel, she said she learned she had “a really horrible calculus teacher” in 1981, who started them with formulas instead of the diagrams. She said she needed to start with a more physical understanding of the concepts—once she had that, she could get the formulas, too. “If I had had this diagram when I was 19, I would have done OK.”

It was interesting to note that all three of the above resolutions shifted some of the blame for failure from the respondents and onto the system that had tried to teach them. Although respondents sometimes personified the issues as their “horrible” calculus teachers, the real problem seemed to be that traditional approaches to teaching calculus had not worked for them. Although *Handling Calculus* was not exactly a second chance to learn calculus, it makes one wonder how these visitors’ academic careers might have differed if their teachers had approached calculus using more kinesthetic, application-oriented, and concept-focused approaches.

SOME FINAL THOUGHTS

By taking a kinesthetic approach to calculus and its precursor concepts, the *Handling Calculus* team developed a series of exhibits that worked, in various ways, for a broad range of visitors. In *Handling Calculus*, math-averse visitors did not need to flee in terror, because making graphs was an enjoyable social experience. By graphing moving bodies and sliders on a track, younger children began to understand motion graphs long before their school curricula dictated. Those still taking school calculus were able to complement their stud-

ies, while successful students from long ago revisited an old friend they thought they had abandoned. Finally, some visitors whose relationship with calculus had hurt them in the past found balms for their old wounds.

In the end, we found the exhibit team was at least partially correct in their original claim that the “kinesthetic experience of physical actions such as motion, shape, and flow can make the ideas of calculus accessible and illuminating to a general audience.” However, based on our evaluations, we would rephrase the team’s initial statement as follows: “By graphing kinesthetic experiences of physical actions, a well-tested exhibit can make a range of *mathematical* ideas accessible and illuminating to a range of museum visitors—up to and including some of the basic principles of calculus.”

The story of *Handling Calculus* continues to unfold even though the summative evaluation is complete. The team continues to remediate the exhibit as staff time and money allow, dealing with issues as straightforward as interface usability and as complex as balancing the needs of young children with those of post-calculus adults. This last issue seems most difficult, in part because the large label panels that were linked to some adults’ most meaningful experiences sometimes distracted parents from the graphics that helped their children make sense of the exhibits. If there is a resolution, it may emerge from the naturally nested concepts that visitors encounter along the road to understanding calculus.

For other museums that want to address complex and sophisticated concepts with visitors, we offer a few lessons learned from our evaluations of *Handling Calculus*. Some lessons crop up time and again in evaluation studies, like the need to set appropriate challenges, address visitors’ existing knowledge, and iteratively test and revise prototype exhibits. Here are some lessons that seem more specific to exhibits about math:

Turn abstract concepts into concrete, kinesthetic experiences. By starting visitors’ interactions with complex, rich, and kinesthetic experiences, the *Handling Calculus* exhibits addressed a range of mathematical concepts that could be understood by visitors with varying levels of mathematical knowledge.

Understand visitors’ complex emotions about math—and find ways to address them in the exhibit. Visitors have a range of emotional connections even to subjects that seem as distant or dry as higher math, and those feelings can shape how they experience your exhibits. Help visitors get past their more negative feelings by presenting them with appropriate challenges, facilitating teaching-learning behaviors, and humor appropriate to the situation.

Put visitors’ fragmentary memories to good use. Tap into visitors’ memories with key words, phrases, and symbols (as identified during front-end evaluation), and then find ways to guide them as they reconstruct concepts they learned long ago, or construct new understandings in your exhibits.

Address complex audiences with nested concepts. Exhibits can better serve multi-age groups when all ages find related forms of meaning in the same experience. Preschoolers, older children, and adults may focus on different aspects of the experience, but the meanings found by younger and less sophisticated visitors can be related or even be prerequisites to

what more sophisticated visitors see and understand. Developers need to find and analyze these meanings first, so they can help parents and caregivers recognize the related concepts and talk with their children in appropriate ways.

Help visitors understand the pedagogy behind the exhibit. A major exhibit message should be that there are many ways to learn math, and the way math is presented in this exhibit may be most appropriate for people like you. If math-averse visitors leave your exhibits thinking about why they dislike math—and how their school-math experiences could have been different—that is a positive outcome.

Based on our evaluations of *Handling Calculus*, we found that an exhibition on higher math can serve a broad and complex audience. We hope that future exhibitions on mathematics can find ways to put our findings to use.

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NOTE

1. The Science Museum of Minnesota is a museum of science and technology in St. Paul, Minnesota. The museum's Experiment Gallery pioneered the concept of Experiment Benches, which provide visitors with the tools and space needed to pursue experiments on a variety of topics. TERC is a not-for-profit education research and development organization oriented towards improving mathematics, science, and technology teaching and learning, primarily through product development. This paper is based on an evaluation contracted by the Science Museum of Minnesota with Selinda Research Associates, Inc., of the exhibits for *Handling Calculus*.

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APPENDIX A

This appendix presents a brief overview of the methodology and methods used in the evaluation studies that form the basis of this paper.

The Science Museum of Minnesota contracted with Selinda Research Associates, Inc., to evaluate the exhibits created for *Handling Calculus*. The evaluation included critical reviews, and formative, remedial, and summative evaluations (Garibay 2002; Garibay and Gyllenhaal 2003b; Gyllenhaal and Garibay 2003; Gyllenhaal and Garibay 2004, Gyllenhaal 2005). All studies were conducted using naturalistic methodology (Lincoln and Guba 1985). Our methods were largely qualitative, including unobtrusive observations, intercept interviews, participant observations, and depth interviews (Miles and Huberman 1994). Respondents for these studies were casual visitors to the Science Museum of Minnesota, purposively selected to address questions laid out in a topical framework developed for each stage of the evaluation.

The findings discussed in this paper emerged from the remedial and summative evaluations. As we worked through these stages, our cumulative sample sizes grew larger than most qualitative studies, reaching more than 300 visitors. (The remedial evaluation study included observations and/or intercept interviews with 113 respondent groups and 284 total respondents. The summative study included more interviews with 32 respondent groups and 55 total respondents.) However, because we sampled visitors purposively (not randomly), in accordance with standards for naturalistic inquiry, we won't make quantitative generalizations about the museum's audience as a whole. When we use terms like "some," "many," or "most" in this paper, we refer only to our purposive sample, not to any larger population.

The interview protocols developed for the remedial and summative stages included questions about the mathematical ideas respondents recalled as they used the exhibits; how they used their memories from school math to make sense of what they saw and did in the exhibits; what they recalled feeling as they used the exhibits; what math courses they had taken in school; their feelings when they recalled these courses; their feelings about mathematics in general; and what they saw as the relevance of math (especially calculus) to their own lives. We tread carefully around visitors' emotional responses to mathematics. Although many respondents spoke quite frankly about their negative feelings—sometimes displaying "I hate math!" as a badge of honor—it was almost painful to talk with others about this topic. In accordance with standards for conducting naturalistic inquiry, we did not pursue questions that made respondents unduly uncomfortable. We usually gained at least some insights into how individual respondents' memories and feelings about school math helped to shape their experiences in the exhibit. When we aggregate these findings, we have a broad picture of the range of visitor experiences with school math and how those influenced visitors' responses to *Handling Calculus*.

Data analysis for this study was a modified inductive constant comparison approach (Lincoln and Guba 1985). We took each new unit of data and systematically compared it to all previous units of data. Categories, patterns, and threads emerged from the data and then were refined, elaborated, or modified as we collected more data and reexamined earlier findings. Analysis began as data were gathered in the exhibition, when preliminary understand-

ings were developed and tested with respondents. Analysis continued through several more levels, including written debriefs after each observation/interview; data summaries at the end of each day; site-visit summaries after each round of data collection; and re-analysis of the full data set when the report was written for that stage of the evaluation.

APPENDIX B

This appendix includes descriptions of the *Handling Calculus* exhibits and label panels discussed in this paper, arranged according to the calculus concepts explored at each exhibit.

Motion graphs and differentiation—Three exhibits explored the making and interpretation of motion graphs and the concept of differentiation. At Motion Tracker, visitors made graphs showing their own bodies' motion over time. As they moved on the yellow rug, a sensor determined their position continuously, and these data were graphed on a computer screen over a 20-second interval (position or velocity on the y-axis, time on the x-axis). Visitors were challenged to move so that they matched a model position graph, or to match a given velocity graph (velocity was calculated as the derivative of visitors' position graphs).

Math Tracks covered similar topics to Motion Tracker, but was designed to encourage much longer and deeper interactions with motion graphs and differentiation. Visitors created and interpreted graphs of the motion of sliders on a track. Instructions on a computer led them through one of three scenarios based on stories, such as a trip to the Post Office or Little Red Riding Hood, and positions of the characters and props were graphed as the story unfolded. Visitors could replay the graph of their story with or without a pre-recorded audio narration, watching the sliders move as their graphs developed on the screen. A Free Exploration option allowed visitors to create free-form motion graphs in a variety of ways. For some stories and the free exploration mode, visitors could create a graph of velocity by having the computer take the derivative of their position graph.

At Road Trip, visitors graphed simulated trips using a computer-controlled car on a vertical track, which corresponded to the vertical y-position axis simultaneously graphed on the monitor. Visitors could recreate a trip that the Science Museum teen volunteers took from St. Paul to Duluth or create their own trips to other cities across the United States. Graphs on the monitor showed a distance versus time plot of the trip, and visitors could display the derivative of that graph, which showed how fast their car was traveling at any time (useful for avoiding speeding tickets from the simulated traffic police).

Slope or "rise-over-run"—Slope Rider allowed visitors to explore the concept of slope, a key to understanding the process of differentiation. In response to challenges posed on a computer monitor, visitors used a rope with a wire-fence grid to measure rise-over-run and calculate the slope of a snowboard with a figure on it. An adjacent label panel explained and illustrated the concept of slope and related it to differentiation.

Integration—At the Flow Integrator, visitors controlled the position of a stream of water over a row of adjacent vertical bins, which served to model the flow of the Mississippi River over a 10-day period. Visitors were challenged to fill the series of bins with just enough water to match one of two curves, one representing the average river flow for each day and the other the development of a flood in the year 2000. Pushing a button released water from the bins into a graduated container to measure the total flow over the interval. This was a mechanical integration of the volume under the curve, as explained and illustrated in an adjacent label panel.

Limits—Exhibits and labels dealing with the key concept of limits were housed in a large gazebo in the center of the exhibit area. The Archimedes' Limit exhibit, in the center of the gazebo, consisted of a circular case that enclosed a laser inside a large-diameter cylindrical mirror that created a sequence of geometrical shapes in a foggy atmosphere. The mechanism turned the laser beam to add sides to the shape—first a triangle, then a square, pentagon, hexagon, and so on—until the myriad sides made the shape resemble a circle. One large label panel on the wall of the gazebo illustrated and explained the process with silhouettes of the sequence of shapes, and another large panel illustrated the relationship between limits and the processes of differentiation and integration.