Handling Calculus:

Graphing Motion to Understand Math

For their 2001–2005 National Science Foundation–funded project Handling Calculus: Math in Motion, *the Science Museum of Minnesota and TERC created a series of interactive exhibits on basic concepts of calculus. In the following article, the co-principal investigator from TERC and the exhibition's evaluator describe how research into the kinesthetics of mathematical understanding informed development of the exhibition and affected the experience visitors had on the floor.*

Mathematics in the Body

By Ricardo Nemirovsky

The notion that thinking is an "embodied" activity—that the active human body as a whole, not just the brain, is involved in how we conceptualize situations—has been developed in the writing of philosophers like Maurice Merleau-Ponty and cognitive scientists like George Lakoff.

This bodily basis of understanding was central to the research on motion and mathematics that would eventually inform the *Handling Calculus* exhibition. But in contrast with other researchers, our intent was not to identify cognitive structures, but to gain insight into how situations involving body motion and symbols look to—and are experienced by—students across levels from elementary school to college.

Many approaches have been developed to account for how people construct and interpret symbolic expressions. Take the situation of someone who traces a line on a paper and refers to "this straight line." The idea that he or she expects to share requires some degree of not seeing the trace, for no actual drawn trace is a geometric straight line. The trace makes pointable a field of linear relations that are the actual subject of the speaker's utterances and gestures.

How then to capture the dynamics of pointing to something that is partly visible and partly imagined? For our studies, we used different types of motion detectors (i.e., computer devices that generate graphs in real time as one moves one's body or an object), as well as other tools that can help people to examine phenomena, such as flow or turning wheels, and symbols, like graphs or number tables, by engaging in bodily activity. We also studied how people move their hands and eyes as they try to make mathematical sense of situations of change.

In our research work, we identified different aspects of what symbol-users experience as they construct and interpret symbolic expressions based on the use of a tool like the motion detector. Our reports centered on the analysis of conversations with students in interview and classroom settings. We participated in these conversations to gain insights not only into how mathematical situations appeared to the students, but also into our own mathematical understandings.

One of the aspects we described was the experience of *fusion*: the blending of action and symbol. Fusion entails merging the qualities of symbols with qualities of the signified events or situations—i.e., talking, gesturing, and envisioning in ways that do not distinguish between symbols and referents. The child who plays "horse" with a stick, for example, is aware that the stick is not a real horse. Yet acting, talking, and gesturing as if the stick were truly a horse is at the essence of his or her ability to play, to be part of a make-believe situation.

Fusion is not exceptional or anomalous. On the contrary, it is ordinary and pervasive. It takes place when we use a map to explain to a friend how to get somewhere, or when we read a poem in which the sound of the words is a crucial aspect to what they come to mean. When fusion occurs, the protagonist enters a bodily space in which the symbols and what they refer to become one. In our study, the students exhibited fusion when they began to speak of the graphs generated by their manipulation of the motion detector in terms of a journey along a particular trajectory.

Another aspect of symbolic understanding is the idea of *dwelling in symbolic spaces.* Think of a room in your home. As you conduct your life in that room, you do not just "interact" with the room; you dwell in it. Dwelling is a bodily phenomenon through which one's surroundings become an extension of the body, and in which the things and others present in those surroundings acquire the qualities of being close or far, to the right or to the left, heavy or light, accessible or hidden, and so forth.

Similarly, fluent symbol-users do not merely interact with symbols drawn on a surface; they dwell in spaces that include them. We call these spaces "symbolic places" or, when they involve mathematical activity, "mathematical places." We see mathematics learning as a process of coming to dwell in mathematical places.

In one of our studies, a student used the motion detector to generate two line graphs simultaneously. She began to swing her arms forward and back in a regular movement, while the computer screen displayed two unfolding graphs of position vs. time, symbolizing the motion of her two arms. Afterward, she told the interviewer, "I thought of getting bigger, bigger, and then smaller, smaller.... I just kind of thought of skiing." Equating this new experience with a familiar bodily experience, she entered a mathematical place that included the graphs of position vs. time displayed before her on the monitor.

Observations like these were the background against which we began to design the exhibit experiences that visitors would encounter in *Handling Calculus.* We wanted to see whether experiences of fusion and graphical space might be possible in the context of a museum exhibition.

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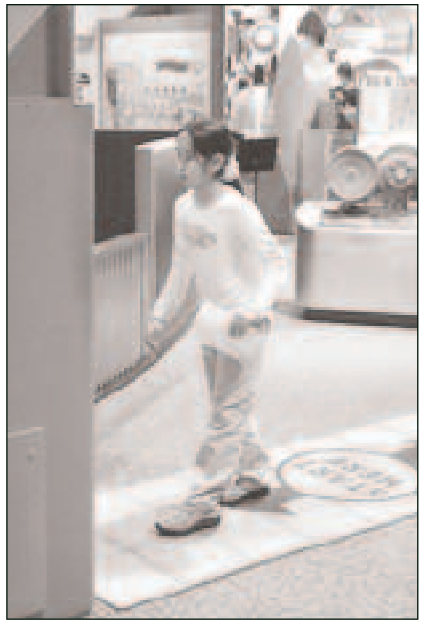
A Different Way to Do Math

By Eric D. Gyllenhaal

The interactive exhibits in *Handling Calculus* linked visitors' physical actions to symbolic expressions—i.e., graphs—on computer monitors. The project team termed this a "kinesthetic" approach to mathematics. Three exhibits explored the making and interpretation of motion graphs and the calculus concept of differentiation. Selinda Research Associates Inc. of Chicago conducted formative, remedial, and summative evaluations of these exhibits, using a naturalistic methodology and qualitative methods, especially conversations with visitors during and after their use of the exhibits.

Each exhibit provided a different opportunity for visitors to experience fusion between their actions and resulting graphs. At Motion Tracker, visitors took the perspective of the tool, making graphs that showed their own bodies' motion over time. As visitors moved back and forth on a long rug, a sensor determined their position continuously, and these data were graphed on the monitor as either a position graph or a velocity graph (the derivative of the position graph). Visitors were challenged to create graphs matching the shapes of on-screen templates, which tested their understanding of their role in the exhibit's graphical space. (Most were much more successful working in position-graph space; velocity-graph space seemed counterintuitive.)



Moving back and forth along the Motion Tracker rug, a visitor watches the graph of her movements unfold on the computer monitor. *Photo by Craig Thiesen/Science Museum of Minnesota*

The evaluators found that users of Motion Tracker had fun and playful experiences. They took away a physical knowledge of what it felt like to make—or, should we say, *be*—a graph, rather than abstract knowledge about graphs and derivatives.

At Math Tracks, visitors also assumed the tool's perspective as they moved sliders along a track and watched the resulting position graphs on screen. Visitors could "fuse" in one of two ways, either assigning roles to the sliders and reenacting stories like "A Trip to the Post Office" or "Little Red Riding Hood," or generating free-form graphs of the sliders' motions. They could then replay and reflect on these graphs, watching the sliders reproduce their earlier actions as the corresponding graphs were redrawn on the monitor.

The evaluators found that Math Tracks, because of its strong kinesthetic approach, use of stories, and opportunities for reflection, successfully engaged visitors and helped them understand motion graphs in new and different ways. Elementary-aged students developed their understanding of what, for them, was a new kind of graph. Older children and adults often developed a gut-level understanding of motion graphs, comparing that favorably with earlier paper-and-pencil learning about graphs.

At Road Trip, visitors designed trips to cities across the United States, making sure to include food and gas stops and avoid speeding tickets. In this case, visitors used the computer as a tool to control a physical object—a car driving on a vertical track. Rather than moving physically to produce a symbol, visitors manipulated the shape of a symbol of the trip—a position graph on the monitor screen—to produce motion in physical space.

The evaluators judged this exhibit less successful at engaging visitors, in part because the more abstract remote control of the car provided a less kinesthetic experience. Perhaps it was harder to achieve fusion when symbols intervened between the visitor's body and the physical motion.

Overall, the *Handling Calculus* team worked hard to achieve this level of success in the three exhibits. These were complex interactives about difficult concepts, and not all issues of usability and understanding were resolved prior to summative evaluation. It was also a challenge to find the terminology to describe what visitors were doing in *Handling Calculus* and the concepts to explain what they were taking away from the experience.

A signal achievement of the exhibition was that many visitors understood that they were engaged in a different approach to math, an approach that was both "more fun" and "understandable in different ways" than traditional school math. For some who had struggled with earlier math experiences, this was both a relief and a revelation. ■

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